$(h/h_0)_\infty$ = ratio of heat transfer coefficients for large Sc and Pr

h_D = mass transfer coefficient, $h_D = \dfrac{D}{W_\infty - W_w}\left.\dfrac{\partial W}{\partial y}\right|_w$

$(h_D/h_{D0})_\infty$ = ratio of mass transfer coefficients for large Sc

k = thermal conductivity

$l(x)$ = $\{3D\int_0^x \beta^{1/2}\,dx\}^{1/3}/\beta^{1/2}$

$\dot{m}_{1\text{diffusion}}$ = surface mass flux of component 1 by diffusion

$\dot{m}_1$ = total surface mass flux of component 1

t = temperature

t_∞ = free stream temperature

u = longitudinal velocity component

$U(x)$ = local free stream velocity

v = transverse velocity component

W = mass fraction of component 1

W_∞ = free stream mass fraction of component 1

x = longitudinal coordinate

y = transverse coordinate

Greek Letters

α = thermal diffusivity, $k/C_p\rho$

β = τ_w/μ for $B = 0$

$\beta_{\text{corr.}}$ = τ_w/μ corrected for effects of surface velocity

η = $y/l(x)$

μ = dynamic viscosity

ν = kinematic viscosity, μ/ρ

ρ = density

τ_w = surface shear stress

N_{Le} = Lewis number, N_{Pr}/N_{Sc}

N_{Pr} = Prandtl number, $\mu C_p/k$

N_{Sc} = Schmidt number, $\mu/\rho D$

Subscripts

0 = condition of $B = 0$

w = conditions at the wall

LITERATURE CITED

1. Mickley, H. S., R. C. Ross, A. L. Squyers, and W. E. Stewart, *Nat'l. Advisory Comm. Aeronaut. Tech. Note 3208* (1954).
2. Merk, H. J., *Appl. Sci. Res.*, **A8**, 237 (1959).
3. Spalding, D. B., and H. L. Evans, *Int. J. Heat Mass Transfer*, **2**, 199, 314 (1961).
4. Acrivos, A., *J. Fluid Mech.*, **12**, 337 (1962).
5. Bird, R. B., W. E. Stewart, and E. N. Lightfoot, "Transport Phenomena", p. 675, Wiley, New York (1960).
6. Lighthill, M. J., *Proc. Roy. Soc. London*, **A202**, 359 (1950)

Equilibrium Flow of a General Fluid Through a Cylindrical Tube

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In a recent communication to the editor (8) questions were raised about the results of work presented in a previously published paper (6). These questions will be answered in the following discussion.

The majority of the questions raised in reference 8 may be dismissed if one accepts the proposition that equilibrium or fully developed flow did exist in the tests reported in reference 6 for a tube with an L/D ratio of about 48, within the range of velocity gradients reported in Figure 4 of the same reference.

In opposition to references 2 and 4, which suggest that the L/D ratio necessary to attain equilibrium flow for viscoelastic fluids must be well over 100, the results of work of several other investigators (1, 3, 5, 7) indicate that equilibrium flow is attained at L/D tube ratios of much less than 48. Furthermore, the L/D ratio required to attain equilibrium flow should be a property of the test material as well as of the existing flow conditions.

The fact that some experimenters have found that σ_1 is smaller than τ does not necessarily contradict the results shown in Figure 4 (6), since at $-\left[\dfrac{dv_z}{dr}\right] < \sim 45$ sec.$^{-1}$, $\sigma_1 < -\tau$. In accordance with this the material function σ_1 should increase more rapidly with increasing velocity gradient than should the material function τ. Thus above some limiting velocity gradient, which will also depend on the specific test material, σ_1 would be expected to be greater in magnitude than τ.

With regard to the boundary condition at the tube exit, $[p_{ro}]_L$, since at $z = L$, and $r = 0$, $\left[\dfrac{dv_z}{dr}\right] = 0$, it follows that $\dfrac{d[p_{ro}]_L}{da} = 0$, although $[p_{ro}]_L$ may not be zero. Equation (18) of reference 6 should be written as

$$[p_{ro}]_L = -[p'']_{r=0,L} = [p_{zL}]_{r=0} \equiv h \qquad (18a)$$

where h is a parameter, which does not depend on the velocity gradient $\left[\dfrac{dv_z}{dr}\right]$. In place of Equations (19) and (20) in reference 6

$$P_{RL} = -[p_{rR}]_L = \int_o^R \xi^{-1}\sigma_1\left[\tau^{-1}\left(\frac{1}{2}a\xi\right)\right]d\xi - h \qquad (19a)$$

and

$$P_{RL} = \int_o^{\frac{1}{2}aR} \xi^{-1}\sigma_1[\tau^{-1}(\xi)]\,d\xi - h \qquad (20a)$$

The significance* of the parameter h can be seen by an evaluation of Equation (22) in reference 6 at $r = R$ and $z = L$ and a comparison with Equation (26) in reference 6 which reads

$$\sigma_2\left[\tau^{-1}\left(\frac{1}{2}aR\right)\right] = \sigma_1\left[\tau^{-1}\left(\frac{1}{2}aR\right)\right] + P_{RL} + \frac{4}{aR^2}\frac{d}{da}\left[\int_o^{\frac{1}{2}aR} p_{zL}\xi\,d\xi\right] \qquad (26)$$

when one finds that

$$\frac{4}{aR^2}\frac{d}{da}\left[\int_o^{\frac{1}{2}aR} p_{zL}\xi\,d\xi\right] = [p_{zL}]_R$$

It can also be shown that when the

* The introduction of the parameter h does not affect any other equations of reference 6, with the exception of Equation (33), nor does it affect the experimental measurements, computations, and conclusions drawn from these measurements. Equation (33) is, of course, valid for $h = 0$.

gravitational force is neglected and $J_m \simeq 0$ is taken

$$[p_{zL}]_R = \frac{J}{A} 1 + \frac{1}{2} \frac{d \ln k_1}{d \ln a} + \frac{k_1}{2} \sigma_1$$

where

$$k_1 = \left(\frac{J/A}{P_{RL}} \right)$$

Thus, if $[p_{Lz}]_R$ is a unique function of the velocity gradient $\left[\frac{dv_z}{dr} \right]_{r=R}$, that is, if it represents uniquely a material property, then one may conclude from Equations (20*a*) and (26) that $h = 0$. The reverse is also true. Also, the magnitude of the parameter h need not be known since P_{RL} is a measured quantity. Since in the tests of reference 6 P_{RL} was found to be much larger than J/A, it follows that $k_1 \simeq 0$ and $h \simeq 0$.

With regard to the assumption made in reference 6 that the fluid is completely relaxed at the point where $A = A_m$, if $F_m \neq 0$, then the magnitude of the material parameter σ_2 is simply larger than computed since P_{RL} is a measured quantity [see Equations (26) and (27) of reference 6]. The accuracy with which the area ratio $\alpha = A_m/A$ is known in the tests of reference 6 is not important since the magnitude of the stress $[p_{zL}]_R$ relative to P_{RL} was determined as an upper limit; that is, for $J_m \to 0$ when α would be very large compared to unity.

NOTATION

A = cross-sectional area, sq. ft.
a = driving force per unit volume in the direction of flow, lb.-force/(sq. ft.)(ft.)
D = diameter, ft.
F = total tensile force, lb.-force
J = momentum flux, lb.-force
L = specified tube length, ft.
P = pressure measured by observer at tube wall, lb.-force/sq. ft.
p = stress component, lb.-force/sq. ft.
p'' = hydrostatic pressure, lb.-force/sq. ft.
R = radius, ft.
r = radial coordinate or radial distance from tube or jet axis, ft.
v_z = velocity component in z direction, ft./sec.
z = axial coordinate or axial distance, ft.
α = A_m/A, area ratio, dimensionless
ξ = dummy variable, appropriate units
σ_1 = material function, lb.-force/sq. ft.
σ_2 = material function, lb.-force/sq. ft.
τ = shear stress, lb.-force/sq. ft.

Subscripts

L = at tube exit
m = at position of maximum jet diameter
o = at tube or jet axis, or at origin of z coordinate
R = at tube wall
r = in the r direction, or at a radial distance, r
z = in the z direction, or at an axial distance, z

LITERATURE CITED

1. Bagley, E. B., *J. Appl. Phys.*, **28**, 624 (1957).
2. Bagley, E. B., S. H. Storey, and D. C. West, *J. Appl. Polymer Sci.*, to be published.
3. Bauer, W. H., and Weber, N., *J. Phys. Chem.*, **60**, 270 (1956).
4. Metzner, A. B., W. T. Houghton, R. A. Sailor, and J. L. White, *Trans. Soc. Rheol.*, **5**, 133 (1961).
5. Philippoff, W., and F. H. Gaskins, *ibid.*, **2**, 263 (1958).
6. Sakiadis, B. C., *A.I.Ch.E. Journal*, **8**, 317 (1962).
7. Schott, H., and W. S. Kaghan, *J. Appl. Polymer Sci.*, **5**, 175 (1961).
8. White, J. L., *A.I.Ch.E. Journal*, **9**, 559 (1963).

Catalytic Effectiveness in Multicomponent and Variable Diffusivity Systems

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A method of evaluating catalytic effectiveness in systems of variable diffusivity is proposed. These systems are encountered when catalysts with bi-disperse pore-size distributions are employed or when multicomponent mixtures are involved. The diffusivity is taken to be a linear function of length, and an effectiveness factor is defined for this case. Application of the method is discussed and a numerical example is given.

The problem of diffusional retardation of reaction kinetics in catalytic systems originally studied by Thiele (*1*) has been extended by numerous workers. Some of the more important of these investigations have dealt with further study of isothermal and non-isothermal systems (*2, 3, 4, 5, 6*) and of the effects of catalyst geometry and pore-size distribution (*7, 8, 9, 10, 11*) on the combined diffusion-reaction process.

It has been shown in this prior work that the effectiveness of a catalyst in carrying out a reaction depends on the mechanism of diffusion which occurs, and that the structure of the catalyst is very important in determining this. In many supported catalysts of commercial importance one of the predominant characteristics of the structure is a dual